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Diffraction by a gap between two breakwaters: solution for long waves by matched asymptotic expansions

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A mathematical model is constructed to represent the diffraction of plane harmonic waves through a gap between two semi-infinite breakwaters in water of constant depth. The boundary-value problem corresponding to this model is formulated and then specialized to the case of waves that are long relative to the gap width. A solution to the long-wave problem is found using the method of matched asymptotic expansions. A selection of results are presented and, where possible, comparisons are made with previous work.

1. Introduction

The problem of water-wave diffraction by a gap between two semi-infinite breakwaters is investigated. The breakwater configuration to be considered, shown in plan view in figure 1, consists of two elements inclined to each other at an angle of $\pi - 2\beta, 0 < \beta < \frac{1}{2}\pi$. The breakwaters are assumed to be infinitesimally thin with vertical walls which are perfectly reflecting. It is further supposed that the configuration is in water of uniform depth h. The fluid motion is taken to be induced by a train of small-amplitude monochromatic plane waves; the case of random waves may then be obtained from the single-frequency results by using Fourier analysis. The standard assumptions are made, that is that the fluid is inviscid, incompressible and homogeneous, and in irrotational motion to which linear theory applies.

The above hypotheses allow the diffraction process to be described by a boundary-value problem in two dimensions. The formulation of the boundary-value problem is given in detail in §2. We then specialize the equations to the case of waves that are long relative to the gap width between the two breakwaters and use the method of matched asymptotic expansions to find a solution. This method of solution has its basis in the physical problem, from where it may be seen that there are two distinct important lengthscales. At points sufficiently far from the breakwater gap it will appear that the fluid flow at the gap is simply due to a source. This is the 'outer' field and a solution found there will be valid at unscaled distances $O(\lambda/(2\pi))$, where λ is the wavelength, from the gap. In the vicinity of the breakwater gap, the 'inner' field, the important lengthscale is the gap width. A solution may be obtained for both the outer and inner problems in the form of an asymptotic expansion in an appropriately scaled variable. To find a solution at all points in the field the two expansions are related by a matching procedure. The approach used in this paper is similar to that used by Tuck (1975). The inner and outer expansions and the matching procedure are discussed in §3. In addition in §3, we discuss the way in which the solution to this problem may be used in practical circumstances. A

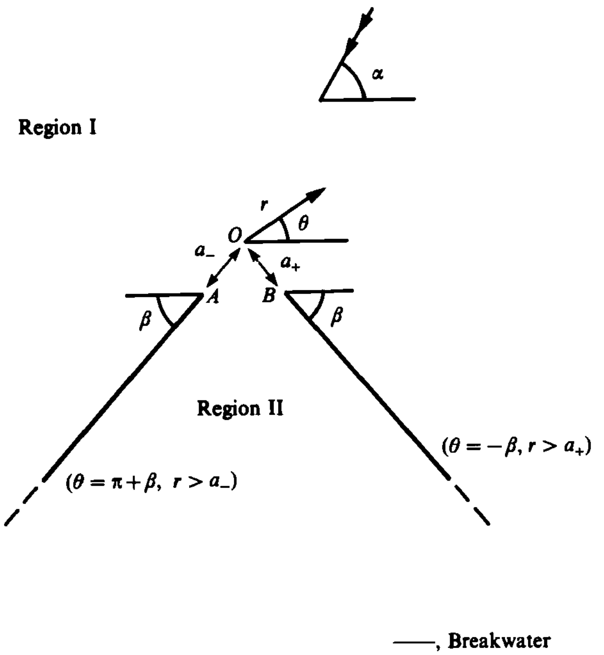

FIGURE 1. Plan view of the breakwater configuration.

selection of results obtained using this method are given in §4, with the conclusions in §5.

The method of matched asymptotic expansions has previously been used by Memos (1980) to find solutions to two special cases of the problem discussed here. These cases correspond to $\beta > 0, OB = 0$, see figure 1, which represents an asymmetric gap and $\beta > 0, OB = OA$, a symmetric gap. Further reference to the solutions of Memos may be found in the appropriate places in the text. It is believed that the application of this method to the general configuration is new.

2. Formulation of the boundary-value problem

A summary of the equations governing the fluid motion are given here. A more detailed account may be found in Wehausen & Laitone (1960).

Under the assumptions given in §1, the velocity field may be expressed in terms of a potential function $\Phi(x,y,z,t)$ by $\boldsymbol{q} = -\nabla\Phi$, where (x,y,z) are Cartesian coordinates with z measured vertically upwards, $z=0$ coinciding with the undisturbed free surface. It is assumed that Φ is periodic in time, with an imposed angular frequency σ. Further, since the breakwaters are supposed vertical and the undisturbed fluid depth h is constant both the vertical and time dependence of the fluid motion can be anticipated and subsequent calculations simplified by setting

$$\Phi = \mathrm{Re}\left[\frac{g}{i\sigma}\frac{\cosh k(z+h)}{\cosh kh}\phi(x,y)\,\mathrm{e}^{-i\sigma t}\right].$$

The wavenumber k is given by the dispersion relation

$$\sigma^2 = gk\tanh(kh).$$

The vertical displacement of the free surface from its equilibrium position is

$$\eta(x, y, t) = \mathrm{Re}\,[\phi(x, y)\,\mathrm{e}^{-\mathrm{i}\sigma t}], \tag{2.1}$$

where $\phi(x, y)$ is a reduced potential satisfying the Helmholtz equation

$$\phi_{xx} + \phi_{yy} + k^2\phi \equiv (\nabla^2 + k^2)\,\phi = 0, \tag{2.2}$$

at all points (x, y) corresponding to the fluid domain.

In terms of the polar coordinates (r, θ), where $x = r\cos\theta, y = r\sin\theta$ the breakwaters lie along the line segments $B_{\pm}$ specified by

$$B_+ : \theta = -\beta, \quad r > a_+,$$
$$B_- : \theta = \pi + \beta, \quad r > a_-,$$

where $\beta > 0$. As the breakwaters are assumed to be perfectly reflecting the boundary conditions may be expressed as,

$$\frac{\partial\phi}{\partial\theta} = 0, \quad \text{on} \quad B_{\pm}. \tag{2.3}$$

From (2.1) $\phi(x, y)$ must be everywhere bounded. Although derivatives of ϕ do not generally exist near the breakwater ends, they must be integrable there (the so-called 'edge' condition). It can be shown that for a thin perfectly reflecting breakwater the derivatives of ϕ near the ends generally exhibit a square-root singularity.

We must also impose the radiation condition

$$\lim_{r\to\infty} r^{\frac{1}{2}}\left(\frac{\partial}{\partial r} - \mathrm{i}k\right)\phi_{\mathrm{d}} = 0, \tag{2.4}$$

uniformly in θ, where $\phi_{\mathrm{d}}(x, y)$, the diffracted potential, is that part of the potential ϕ remaining when all parts representing plane wavetrains (incident and reflected) have been removed. The edge condition and (2.4) guarantee a unique solution provided the fluid motion is induced by an assigned incident wavetrain, which may be represented by

$$\phi_{\mathrm{i}}(r, \theta) = \exp\left(-\mathrm{i}kr\cos(\theta - \alpha)\right), \tag{2.5}$$

the direction of the incident wave making an angle α with the x-axis, as shown in figure 1, where $-\beta < \alpha < \pi + \beta$.

3. The method of matched asymptotic expansions

3.1. *The outer problem*

In order to find the outer solution of the problem we first non-dimensionalize the variables by setting

$$R = kr.$$

The Helmholtz equation (2.2) becomes

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\phi}{\partial\theta^2} + \phi = 0 \quad \text{in the fluid domain,}$$

and the boundary conditions (2.3) are rewritten as

$$\frac{1}{R}\frac{\partial \phi}{\partial \theta} = 0 \quad \text{on} \quad \begin{cases} \theta = \pi+\beta, & R > \kappa_-, \\ \theta = -\beta, & R > \kappa_+, \end{cases}$$

where $\kappa_- = \kappa a_-$ and $\kappa_+ = \kappa a_+$.

It may be shown that the potential in the field may be decomposed as

$$\phi(R,\theta) = \begin{cases} \dfrac{2\pi\hat{a}}{(\pi+2\beta)} \sum\limits_{m=0}^{\infty} \epsilon_m \cos\mu_m(\theta+\beta)\cos\mu_m(\alpha+\beta) J_{\mu_m}(R)\, \mathrm{e}^{-\frac{1}{2}\mathrm{i}\mu_m \pi} \\ \qquad\qquad +\phi_{\mathrm{D}}(R,\theta), \quad -\beta < \theta < \pi+\beta, \\ \phi_{\mathrm{D}}(R,\theta) \qquad\qquad \pi+\beta < \theta < 2\pi-\beta. \end{cases}$$

Here $\hat{a}$ is the incident wave amplitude, $\mu_m = m\pi/(\pi+2\beta)$ and $\epsilon_m = 2, m \geqslant 1, \epsilon_0 = 1$. In the above expression the summation term corresponds to the diffraction of plane waves by a wedge of angle $(\pi-2\beta)$ with perfectly reflecting faces on $\theta = \pi+\beta, \theta = -\beta$, see Smallman & Porter (1985). The potential $\phi_{\mathrm{D}}(R,\theta)$ is the diffracted potential due to the presence of the gap which satisfies the radiation condition

$$R^{\frac{1}{2}}\left(\frac{\partial}{\partial R} - \mathrm{i}\right)\phi_{\mathrm{D}} \to 0 \quad \text{as } R \to \infty,$$

uniformly in θ. The edge condition, discussed in §2, may also be similarly non-dimensionalized.

For waves that are long relative to the gap width, so that $\kappa_+, \kappa_- \ll 1$, a solution for $\phi(R,\theta)$ is sought in the form of an asymptotic expansion. The small parameters κ_+ and κ_- are both ratios of two lengthscales and therefore one expects that the expansion will not be uniformly valid. A second expansion for the problem in independent variables for the inner field will be required to complement it and this will be considered further in §3.2.

As $\kappa_-, \kappa_+ \to 0$ the breakwater gap reduces to a slit of negligible width at the origin. In this limit the flow can be represented by a source at $R = 0, \theta = 0_+$ and a sink at $R = 0, \theta = 0_-$. The imposed requirements on ϕ are satisfied by

$$\phi(R,\theta) = \begin{cases} \dfrac{2\pi\hat{a}}{(\pi+2\beta)} \sum\limits_{m=0}^{\infty} \epsilon_m \cos\mu_m(\theta+\beta)\cos\mu_m(\alpha+\beta) J_{\mu_m}(R)\, \mathrm{e}^{-\frac{1}{2}\mathrm{i}\mu_m} \\ \quad + \dfrac{M\pi}{(\pi+2\beta)} H_0^{(1)}(R), \quad -\beta < \theta < \pi+\beta, \\ \quad - \dfrac{M\pi}{(\pi-2\beta)} H_0^{(1)}(R), \quad \pi+\beta < \theta < 2\pi-\beta, \end{cases} \tag{3.1}$$

where M is a source strength, which is as yet unknown. Expressions (3.1) are valid where R is $O(1)$ with respect to κ_+ and κ_-; that is in the outer field at unscaled distances $O(k^{-1})$ from the gap. That (3.1) is inappropriate near the gap can be seen by noting that, as $R \to 0$,

$$H_0^{(1)}(R) = \frac{2\mathrm{i}}{\pi}\log\left(\tfrac{1}{2}CR\right) + 1 + O(R^2 \log R),$$

where $\log C = \gamma$, the Euler constant. Hence as $R \to 0$ from (3.1)

$$\phi(R,\theta) = \begin{cases} \dfrac{2\pi\hat{a}}{(\pi+2\beta)} + \dfrac{M\pi}{(\pi+2\beta)}\left\{1+\dfrac{2\mathrm{i}}{\pi}\log\left(\tfrac{1}{2}CR\right)\right\} + O(R), & -\beta < \theta < \pi+\beta, \quad (3.2a) \\ \dfrac{-M\pi}{(\pi-2\beta)}\left\{1+\dfrac{2\mathrm{i}}{\pi}\log\left(\tfrac{1}{2}CR\right)\right\} + O(R^2\log R), & \pi+\beta < \theta < 2\pi-\beta, \quad (3.2b) \end{cases}$$

which reveals the non-uniform nature of the solution as $R \to 0$. On physical grounds the non-uniformity arises because near the gap the important lengthscale is the gap width and not $O(k^{-1})$. Expression (3.2) is the inner limit of the outer solution.

3.2. *The inner problem*

In the inner field R is $O(\kappa)$ and in order to examine the flow in the vicinity of the gap the variable R must be stretched and replaced by

$$\hat{R} = \frac{R}{\kappa_-} = \frac{r}{a_-}, \tag{3.3}$$

where it may be assumed without loss of generality that $\kappa_- \geqslant \kappa_+$. In terms of the new variable, the Helmholtz equation (2.2) becomes

$$\frac{1}{\hat{R}}\frac{\partial}{\partial \hat{R}}\left(\frac{\hat{R}\,\partial\phi}{\partial\hat{R}}\right) + \frac{1}{\hat{R}^2}\frac{\partial^2\phi}{\partial\theta^2} + \kappa_-^2\,\phi = 0 \quad \text{in the fluid domain,}$$

and the boundary conditions (2.3) are rewritten as

$$\frac{1}{\hat{R}}\frac{\partial\phi}{\partial\theta} = 0 \quad \text{on} \quad \begin{cases} \theta = -\beta, & \hat{R} > \epsilon, \\ \theta = \pi+\beta, & \hat{R} > 1, \end{cases} \tag{3.4}$$

where $\epsilon = \kappa_+/\kappa_- \,(= a_+/a_-) \leqslant 1$. We assume that $\epsilon = O(1)$ on the scale of κ_+ (a restriction that will be relaxed later), i.e. that κ_+ and κ_- are of the same order.

It can be seen that the leading-order term in the asymptotic expansion of $\phi(\hat{R},\theta)$ satisfies

$$\frac{1}{\hat{R}}\frac{\partial}{\partial \hat{R}}\left(\frac{\hat{R}\,\partial\phi}{\partial\hat{R}}\right) + \frac{1}{\hat{R}^2}\frac{\partial^2\phi}{\partial\theta^2} = 0 \quad \text{in the fluid domain,} \tag{3.5}$$

rather than the Helmholtz equation. This simplifies the problem greatly, in that near the gap potential-flow techniques may be used to find a solution.

A solution of (3.5) is required that satisfies the boundary conditions (3.4) and is suitable from the point of view of matching. The most direct method of finding a solution is via a conformal mapping of the fluid domain in $(\hat{R},\theta)$ coordinates to the upper half-plane in (u,v) coordinates. In order to do this we introduce the complex unit $\mathrm{j} = \sqrt{-1}$ which does interact with the complex unit i already in use. We define

$$\hat{Z} = \hat{R}\,\mathrm{e}^{\mathrm{j}\theta}, \quad \omega = u + \mathrm{j}v, \tag{3.6}$$

the required mapping is given by,

$$\hat{Z} = \frac{\mathrm{e}^{\mathrm{j}(2\pi-\beta)}}{\pi\omega^{q/\pi}}\frac{1}{b^{2\beta/\pi}}\frac{[2\beta q\omega^2 - (1-b)pq\omega + b2\beta p]}{[p-bq]}, \quad 0 < \beta < \tfrac{1}{2}\pi, \tag{3.7}$$

where $p = \pi + 2\beta$ and $q = \pi - 2\beta$,

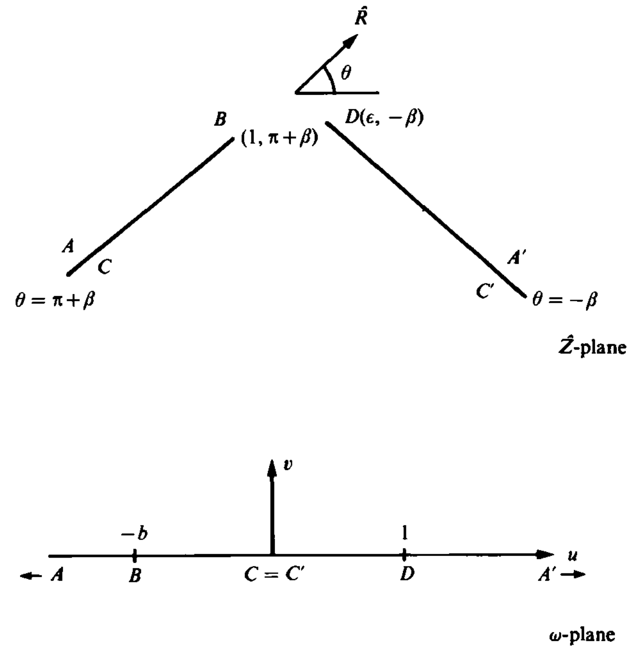

FIGURE 2. Details of the conformal map.

and the positive parameter b is given by

$$\epsilon = \frac{[bp-q]}{b^{2\beta/\pi}[p-bq]}. \tag{3.8}$$

This transformation is of the Schwartz–Christoffel type and is shown in plan in figure 2. It should be noted that the mapping (3.7) is only valid for $\epsilon \geqslant 0$. A mapping of a similar configuration to the upper half-plane appears in Kober (1957). The points at infinity on the exposed and sheltered sides of the breakwater map onto the point at infinity in $v > 0$, and $\omega = 0$ respectively. The breakwater boundary maps onto the u-axis. The boundary conditions (3.4) together with matching considerations lead to the solution

$$\phi(R, \theta) = \begin{cases} \mathrm{Re}_{\mathrm{j}}(m \log \omega + D), & -\beta < \theta < \pi+\beta, \\ \mathrm{Re}_{\mathrm{j}}(m \log \omega + D), & \pi+\beta < \theta < 2\pi-\beta, \end{cases} \tag{3.9}$$

where m, which is real with respect to j, and D are unknown constants.

In order to match the two asymptotic expansions we require the outer limit of the inner solution (3.9). To find the outer limit of (3.9) we observe from (3.7) that

(i) as $$|\omega| \rightarrow \infty \quad (\text{i.e. } |\hat{Z}| \rightarrow \infty)$$

$$\hat{Z} \sim \frac{\mathrm{e}^{\mathrm{j}(2\pi-\beta)} 2\beta q \omega^{p/\pi}}{\pi[p-bq]\, b^{2\beta/\pi}}, \quad \beta > 0$$

$$\therefore \quad \phi = \frac{m\pi}{p} \log\left(\frac{\hat{R} b^{2\beta/\pi}[p-bq]\,\pi}{2\beta q}\right) + D', \tag{3.10}$$

where $D' = \mathrm{Re}\,(D)$.

(ii) as $|\omega| \to 0$ (i.e. $|\hat{Z}| \to \infty$)

$$\hat{Z} \sim \frac{e^{j(2\pi-\beta)} 2\beta p b^{q/\pi}}{\pi[p-bq]\,\omega^{q/\pi}},$$

$$\phi = -\frac{m\pi}{q} \log\left(\frac{\hat{R}[p-bq]\,\pi}{b^{q/\pi} 2\beta p}\right) + D'. \tag{3.11}$$

Expressions (3.10) and (3.11) are the outer limits of the inner solution to the problem.

3.3. *The matching procedure*

In order to complete our solution the unknown constants M, m and D must be found via the matching procedure. In order to do this we match the inner limit of the outer solution with the outer limit of the inner solution having first written both in common variables (r, θ) using (3.3). Matching (3.10) with (3.2a) and (3.11) with (3.2b) yields the following values for the constants:

$$M = -\frac{\hat{a}q}{\pi}\left\{1 + \frac{2i}{\pi} \log \frac{C\kappa_- \beta b^{q/2\pi} q^{q/2\pi} p^{p/2\pi}}{\pi[p-bq]}\right\}^{-1},$$

$$m = \frac{2i}{\pi} M$$

and

$$D' = \hat{a} + \tfrac{1}{2} m \log \frac{q}{bp}. \tag{3.12}$$

Having found the unknown constants M, m and D' the potential at all points in the field may be recovered by substitution of the constants in expressions (3.2), (3.10) and (3.11). However for present purposes the main aim is to find the far-field diffraction coefficients, which are of particular interest to coastal engineers. The far-field diffraction coefficient is obtained from the leading-order term of the diffracted potential, $\phi_D(r, \theta)$ as $r \to \infty$. The amplitude of this leading-order term may be written as $(2\pi r)^{-\frac{1}{2}}|(F(\alpha, \theta)|$, where r is the (dimensionless) distance from the origin. $F(\alpha, \theta)$ is the far-field diffraction coefficient, which is dependent upon α, the incident-wave direction and also upon the particular breakwater arrangement. In many cases coastal engineers require the ratio of the diffracted wave height at a point (r, θ) in the field some distance from the breakwater tip to the height of a uniform incident plane wave. If the plane wave is of unit amplitude then this ratio $K(r, \theta)$ may be expressed as

$$K(r, \theta) = (2\pi r)^{-\frac{1}{2}}|F(\alpha, \theta)|,$$

so that $F(\alpha, \theta)$ need only be evaluated once for any θ and particular breakwater arrangement. Hence, given tables of values of the far-field diffraction coefficient for several values of θ and for a specific breakwater configuration, the diffracted wave-height ratio $K(r, \theta)$ can be calculated at any point (r, θ) which is sufficiently far from the breakwater tip. Therefore, in the case of a breakwater in water of uniform depth, the far-field coefficients give the wave heights at points in the flow field far from the breakwater ends. In addition, the far-field coefficients for a breakwater in water of uniform depth constitute the initial data for solving the diffraction/refraction problem for the same breakwater configuration in water of varying depth. Further information on how this procedure is carried out is given by Southgate (1985).

3.4. *Far-field behaviour*

In order to obtain an expression for the far-field diffraction coefficient we consider the behaviour of (3.1) as $R \to \infty, \pi+\beta < \theta < 2\pi-\beta$, which represents the sheltered part of the harbour in the idealized problem. From (3.1), noting the following (see Abramowitz & Stegun 1965):

$$H_0^{(1)}(R) = \left(\frac{2}{\pi R}\right)^{\frac{1}{2}} e^{i(R-\frac{1}{4}\pi)}\left(1+O\left(\frac{1}{R}\right)\right),$$

we have

$$\phi(R,\theta) = \frac{\hat{a}}{(2\pi R)^{\frac{1}{2}}} e^{i(R+\frac{5}{4}\pi)} F_{\mathrm{II}}(\alpha,\theta)\left(1+O\left(\frac{1}{R}\right)\right), \quad \pi+\beta < \theta < 2\pi-\beta,$$

as $R \to \infty$. Here $F_{\mathrm{II}}(\alpha,\theta)$ is the far-field diffraction coefficient for long waves,

$$F_{\mathrm{II}}(\alpha,\theta) = -\frac{2i\pi}{(\pi-2\beta)}\frac{M}{\hat{a}}.$$

By substituting for M from (3.12) F_{II} may be written as

$$F_{\mathrm{II}}(\alpha,\theta) = 2i\left\{1+\frac{2i}{\pi}\log\frac{C\kappa_-\beta b^{q/2\pi}q^{q/2\pi}p^{p/2\pi}}{\pi[p-bq]}\right\}^{-1}. \tag{3.13}$$

The relation between the ratio of the gap widths ϵ and the far field may be regarded as being given parametrically in terms of b by (3.8) and (3.13).

Several comments may be made about (3.13). The first is that (3.13) is for $\kappa_+, \kappa_- \ll 1$, where κ_+ and κ_- are the dimensionless gap widths, which are related to the actual gap widths, a_+ and a_-, via κ_+ and $\kappa_- = ka_-$ where $k(=2\pi/\lambda)$ is the wavenumber. Results will be given for gap widths a_+ and a_- that are measured in wavelengths λ. It should also be noted that (3.13) is independent of the incident angle α and the observation angle θ and depends only upon gap widths κ_+ and κ_- (through the ratio ϵ) and the breakwater angle β.

Also, it should be observed that the conformal map (3.7), and hence the expression (3.13), is only valid for $0 < \beta < \frac{1}{2}\pi$. For the special case $\beta = 0$ the counterpart of (3.13) can be easily derived and may be shown to be

$$[F_{\mathrm{II}}(\alpha,\theta)]_{\beta=0} = 2i\left\{1+\frac{2i}{\pi}\log\left(\tfrac{1}{8}C\kappa_-(1+\epsilon)\right)\right\}^{-1}. \tag{3.14}$$

The far-field coefficients for the case $\beta < 0$ could also be derived using similar techniques to those described here for $\beta > 0$, but, as this is less significant from a physical point of view it is not included here.

For the special case of the breakwater arrangement with $a_+ = 0$ (one-sided gap) it is possible to use a similar conformal-mapping technique to obtain an expression for the far-field coefficient. The details of the mapping for this special case are given in Smallman (1983) where it is shown that the far-field coefficient is

$$F'_{\mathrm{II}}(\alpha,\theta) = 2i\left\{1+\frac{2i}{\pi}\log\left[\frac{C\kappa_-}{8\pi^2}p^{p/\pi}q^{q/\pi}\right]\right\}^{-1}. \tag{3.15}$$

From (3.8) it can be seen that as $b \to q/p(b \to (\pi-2\beta)/(\pi+2\beta)), \epsilon \to 0(\kappa_+ \to 0)$ and that (3.15) is correctly given in the limit. This implies that, although $\epsilon = O(1)$ has been

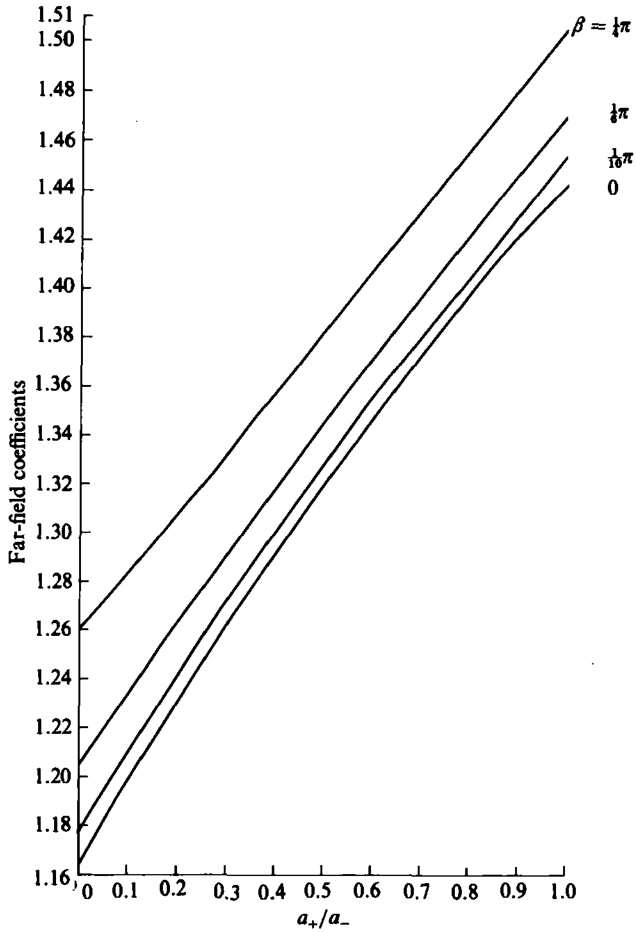

FIGURE 3. Far-field coefficient as a function of gap-width ratio (a_+/a_-) for $a_- = 0.08$ wavelengths ($\kappa_- = 0.5$).

assumed in the derivation of (3.13), the latter is nevertheless uniformly valid as $\epsilon \to 0$. The expression (3.15) coincides with that given by Memos (1980) for the same breakwater configuration.

4. Results

By assigning values to the parameter b it is possible to exhibit graphically the relation between F_{II} and ϵ given by (3.8) and (3.13). Before doing this, some comment may be made on the range of values of b. From (3.8) it can be seen that

$$\frac{q}{p} \leqslant b \leqslant 1 \Rightarrow 0 \leqslant \epsilon \leqslant 1.$$

In particular if $b = q/p = (\pi - 2\beta)/(\pi + 2\beta)$ then $\epsilon = 0$ and the general configuration becomes the special case discussed where $a_+ = 0$, the one-sided gap. Similarly if $b = 1$ then $\epsilon = 1$ which is the special case of the symmetric configuration, $a_+ = a_-$.

Several calculations were made for $|F_{II}|$ using (3.8) and (3.13) for various values of the parameter b. The far-field coefficient $|F_{II}|$ was plotted against the ratio ϵ for

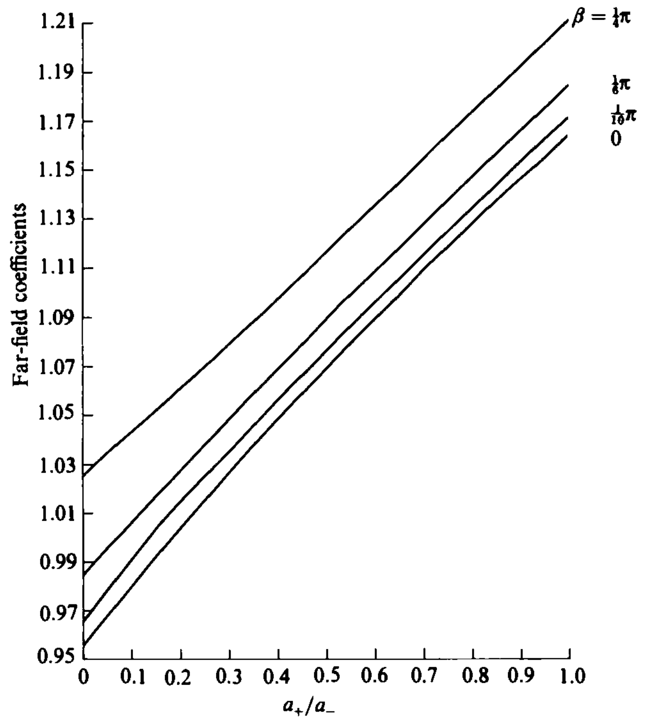

FIGURE 4. Far-field coefficient as a function of gap-width ratio (a_+/a_-) for $a_- = 0.04$ wavelengths ($\kappa_- = 0.25$).

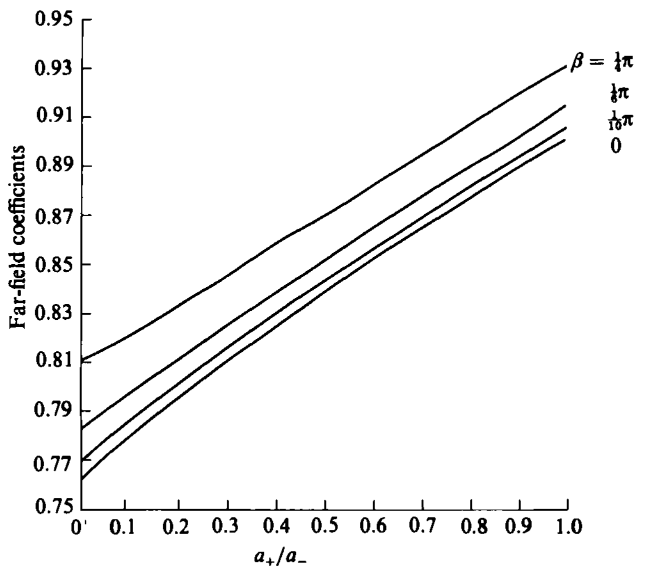

FIGURE 5. Far-field coefficient as a function of gap-width ratio (a_+/a_-) for $a_- = 0.016$ wavelengths ($\kappa_- = 0.1$).

	$\lvert F_{II}\rvert$	
a_-	Present method	Method of Gilbert & Brampton†
0.016	0.90	0.90–0.91
0.040	1.16	1.07–1.23
0.080	1.45	1.13–1.63

† Results for incident angles α/rad, $0 < \alpha < \frac{1}{2}\pi$, observation angles θ/rad, $\pi < \theta < 2\pi$.

TABLE 1. Far-field coefficients $\lvert F_{II}\rvert$ for breakwater angle $\beta = 0$ and gap widths a_-/wavelengths, $\epsilon = 1$ ($a_- = a_+$)

$\kappa_- = 0.1, 0.25, 0.5$ (corresponding to $a_- = 0.1/2\pi, 0.25/2\pi, 0.5/2\pi$ wavelengths) and $\beta = \frac{1}{10}\pi$, $\frac{1}{6}\pi$ and $\frac{1}{4}\pi$. These results are displayed in figures 3, 4 and 5, together with results for $\beta = 0$ from (3.14). From these figures several trends of behaviour of the far-field diffraction coefficients may be noted.

First, as β increases, and therefore the angle between the breakwaters decreases, the value of the far-field coefficient increases for all ϵ. For any fixed gap-width ratio ϵ, as β increases the value of the far-field coefficient lies within a small range, although this range decreases as ϵ increases. It can also be seen from figures 3–5 that for fixed β and a_- the values of the far-field coefficients increase as ϵ increases. This corresponds to the values of the far-field coefficients increasing as the gap on one side of the structure is held fixed and the gap on the other side is increased.

A direct comparison may be made between the results from the present method and those of Gilbert & Brampton (1985) for the special case $\beta = 0$. The method employed by Gilbert & Brampton to calculate the far-field coefficients uses an integral-equation formulation of the problem of diffraction by a gap in a straight breakwater. The integral equations are solved numerically and the method is valid for all gap widths. For the situation where the breakwater gap is not small relative to the incoming wavelength, the far-field coefficients will be dependent on both the incident wave angle α and the obervation angle θ. Therefore for a specified gap width the far-field coefficients will cover a range of values that reflects this dependence. A comparison between the results from the two methods is shown in table 1. It can be seen from the results of Gilbert & Brampton that as the gap width decreases the range of values covered by the far-field coefficients also decreases. The far-field coefficients calculated using the present method are inside the range of those calculated using Gilbert & Brampton's method, and, as expected, the agreement between the two sets of results is at its best for the narrower gap widths.

A qualitative comparison can also be made between the results obtained using a variational method (valid for all gap widths) and those from the present method for the case of the symmetric configuration ($\epsilon = 1$). Such a comparison, for $\beta = \frac{1}{6}\pi$ and $\beta = \frac{1}{4}\pi$, is given in tables 2 and 3. The derivation and use of a variational method in finding the far diffracted field for the general breakwater configuration is given in Smallman & Porter (1985). A similar comment as was made for the results of Gilbert & Brampton applies to the results in the case $a_- = 0.125$ given in tables 2 and 3, which are those calculated using the variational method; that is for a specified gap width the far-field coefficient will cover a range of values that reflects their dependence on the incident-wave angle α and observation angle θ in tables 2 and 3. In fact it is shown in Smallman (1983) that, for the general breakwater configuration,

a_-	$\lvert F_{II}\rvert$
0.016	0.91
0.040	1.18
0.080	1.47
0.125	0.98–2.00†

† Result calculated using the variational method for incident angle $\alpha = \frac{1}{2}\pi$/rad, observation angles θ/rad, $\frac{7}{6}\pi < \theta < \frac{11}{6}\pi$.

TABLE 2. Far-field coefficients $\lvert F_{II}\rvert$ for breakwater angle $\beta = \frac{1}{6}\pi$ rad and gap width a_-/wavelengths, $\epsilon = 1$ $(a_- = a_+)$

a_-	$\lvert F_{II}\rvert$
0.016	0.93
0.040	1.21
0.080	1.51
0.125	2.06†

† Result calculated using the variational method for incident angle $\alpha = \frac{1}{2}\pi$ rad, observations angle θ/rad, $\frac{5}{4}\pi < \theta < \frac{7}{4}\pi$.

TABLE 3. Far-field coefficients $\lvert F_{II}\rvert$ for breakwater angle $\beta = \frac{1}{4}\pi$ rad and gap width a_-/wavelengths, $\epsilon = 1$ $(a_- = a_+)$

as the gap widths narrows the far-field coefficient covers a smaller range of values until we arrive at the case considered here where, for waves that are long relative to the gap width, F_{II} is independent of α and θ. This was discussed in §3.4. It can be seen from tables 2 and 3 that the results from the two methods of solution fit together well and that for small gap widths the trend of the far-field coefficient increasing with increasing gap width is preserved.

5. Conclusions

The method of matched asymptotic expansions has been used to find a solution to the problem of diffraction of long waves by a gap between two breakwaters. In particular, an expression has been found for the far-field diffraction coefficient in the lee of the breakwaters. This coefficient may be used to find the diffracted wave-height ratio at distances far from the breakwater tip and also constitutes the initial data to solve the corresponding diffraction/refraction problem. The far-field diffraction coefficient has been used here to demonstrate trends in the behaviour of the diffracted field for a number of different breakwater configurations.

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